Convergence Analysis on Orthogonal Low Rank Approximation of Tensors

Problem setting

- Determine unit vectors every u_r and λ_r to minimize

$$\left\| T - \sum_{r=1}^{R} \lambda_r \mathbf{u}_r^{(1)} \circ \cdots \circ \mathbf{u}_r^{(k)} \right\|_F^2 .$$

- Best rank-R (R > 1) approximation for high-order tensors may not exist
- Consider orthogonal low rank approximation
- Including completely orthogonal and semi-orthogonal low rank approximation

Algorithm

- SVD is involved for the first $k-\mu$ factors

for $r = 1, 2, \ldots, R$,

$$C^{(\ell)}_{r,[p+1]} = T \circledast_{\beta_\ell} \left(\bigotimes_{i=1}^{\ell-1} \mathbf{u}^{(i)}_{r,[p+1]} \circ \bigotimes_{i=\ell+2}^{k} \mathbf{u}^{(i)}_{r,[p]} \right)$$

$$[\mathbf{u}, s, \mathbf{v}] = \mathrm{svds}(C^{(\ell)}_{r,[p+1]}, 1)$$

end for

- Polar decomposition is applied for last μ factors

for $r = 1, 2, \ldots, R$, **do**

$$\mathbf{v}^{(\ell)}_{r,[p+1]} = T \circledast_{\ell} \left(\bigotimes_{i=1}^{\ell-1} \mathbf{u}^{(i)}_{r,[p+1]} \circ \bigotimes_{i=\ell+1}^{k} \mathbf{u}^{(i)}_{r,[p]} \right)$$

$$\hat{\lambda}^{(\ell)}_{r,[p+1]} := \langle \mathbf{v}^{(\ell)}_{r,[p+1]}, \mathbf{u}^{(\ell)}_{r,[p]} \rangle$$

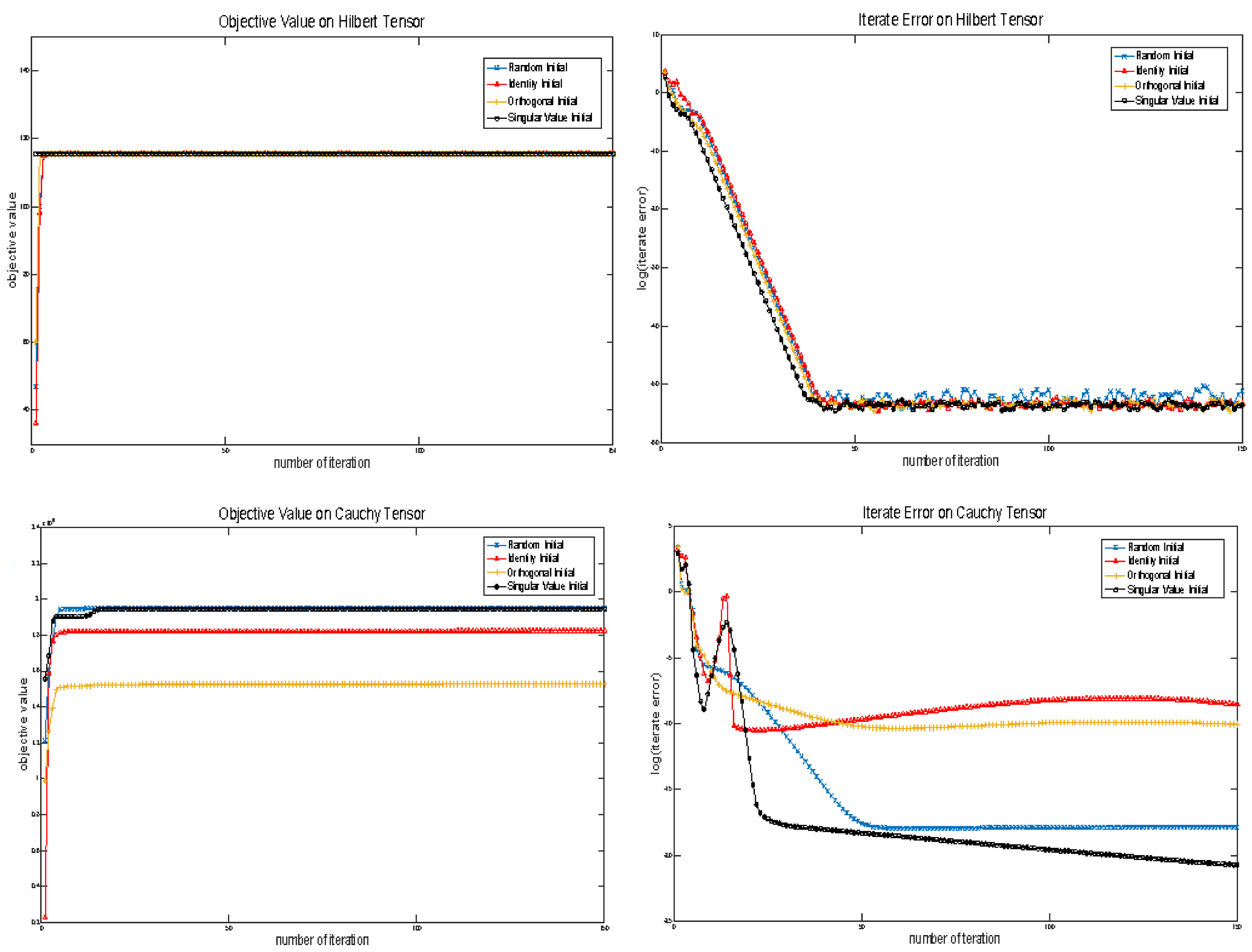
end for

$$[U^{(\ell)}_{[p+1]}, S^{(\ell)}_{[p+1]}] = \mathrm{poldec}(V^{(\ell)}_{[p+1]} \Lambda^{(\ell)}_{[p+1]})$$

Convergence

- the generalized Rayleigh quotients are bounded and monotone increasing
- iterates globally converge to local minimum

Numerical result



Reference

[1] J. CHEN AND Y. SAAD, On the tensor SVD and the optimal low rank orthogonal approximation of tensors, SIAM J. Matrix Anal. Appl., 30 (2008/09), pp. 1709–1734.

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